UDC 621.822.6

ENGINEERING ANALYSIS OF SINGLE ROW THRUST AND ANGULAR CONTACT ROLLING ELEMENT BEARINGS AT COMPLEX LOADING

S.P. Yelizarov, cand.tech.sci.
Odessa State Agrarian University
A.P. Lipin, cand.tech.sci.
Odessa National Academy of Food Technologies

Relationships are obtained for fast and accurate determination of rings displacements as well as a load distribution within separate single row thrust and angular contact rolling element bearings.

Key words: rolling element bearing, load distribution, thrust bearing, angular contact bearing, zone of loading, ring axial and radial displacements, contact angle, iteration process, contact reaction, contact deformation.

Introduction. Calculation of loads acting on elements of any bearing arrangement is a basic stage of its analysis, and the more accurate the load distribution between bearings rolling elements is found the more accurate the method of analysis is considered to be. The distribution mentioned can be found with needed accuracy if external forces and moments acting on every bearing in the arrangement are known. To determine them, a set of equations should be used that takes into consideration external loading factors and that of stiffness of the bearings and the shaft supported.

The problem. It's impossible to find a bearing life accurately if a load distribution between its elements is unknown.

Analysis of recent researches and publications. Basically, up-to-date manuals, reference and study-scientific books include simplified relationships for computation of bearings, and they often give too rough estimates. On the other hand, application of precise methods of calculation requires having special computer programs, specially trained personnel available and considerable time to spend.

The purpose of researches: On the basis of known set of equations for multi-support bearing arrangement analysis, determine relationships for calculation of a load distribution and mutual rings displacements in a separate single row thrust (angular contact) rolling element bearing at its complex loading, with needed accuracy and no large period of time to spend and no special computer knowledge required.

The results of researches: Calculation of a load distribution as well as rings displacements to take place in a separate single row thrust and angular contact rolling element bearing has been made on the basis of known set of equations for multi-support bearing arrangement analysis at any external forces and moments applied, by means of its curtailment to the level of a separate bearing.Set of equations shown in [1] can be used for static analysis of separate rolling element bearings. Let's consider several basic bearing types when assuming a bearing geometry to be ideal and neglecting its rings distortion.

1. **Single row thrust ball bearing loaded with axial off-center force F_z.**

In this case, according to Fig. 1, set of equations [1] comes to two ones:

$$\begin{cases} \Phi_3 = C_\delta \cdot \sum \sigma_{33}^{(k)} \cdot \delta_{zk}^{1.5} - F_z = 0, \qquad \delta_{zk} = f_z + 0.5 \cdot d \cdot \varphi_x \cos\Psi_k; \\ \Phi_4 = 0.5 \cdot C_\delta \cdot d \cdot \sum \sigma_{33}^{(k)} \cdot \delta_{zk}^{1.5} \cdot \cos\Psi_k - M_x = 0 \end{cases} \quad (1)$$

where d - the bearing mean diameter;

$\sigma_{ij}^{(k)}$ – coefficients depending on coordinate angles of rolling elements Ψ_k [1].

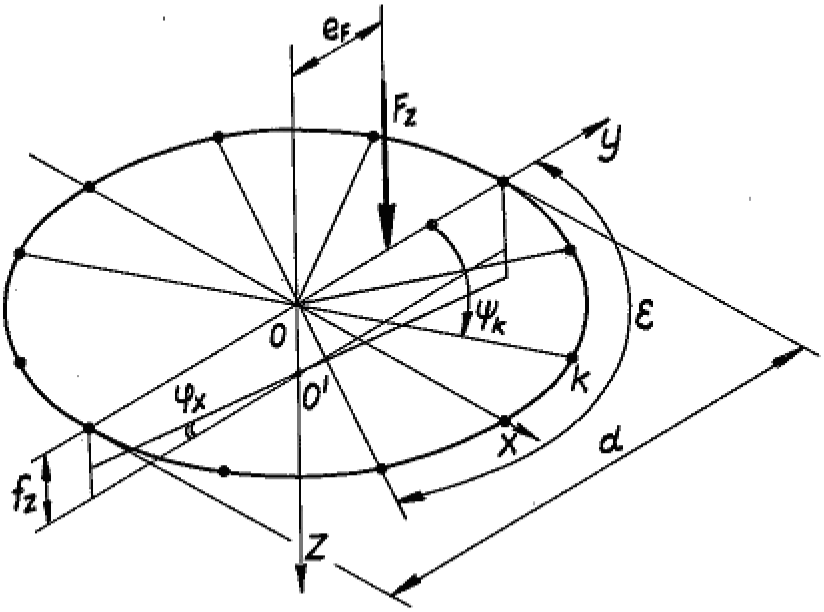

Fig. 1. Loading of a thrust bearing.

Supposing angle $\beta_k = \pi/2$ and reaction $P_{zk} = C_{\delta k} \cdot \delta_{zk}^{1.5} = 0$, one gets a relationship known for determining angle zone ε of the bearing loading:

$$\varepsilon = \arccos\left(-\frac{2 \cdot f_z}{\varphi_x \cdot d}\right).$$

Displacements f_z, φ_x can be found by solving the set (1) with use e.g. of the Newton's method. Coefficients for getting corrections $\Delta f_z, \Delta\varphi_x$ are to be determined using the formulas:

$$a_{33} = 1.5 \cdot C_\delta \cdot \sum \delta_{zk}^{0.5}, \quad a_{34} = a_{43} = 0.75 \cdot C_\delta \cdot d \cdot \sum \delta_{zk}^{0.5} \cdot \cos\Psi_k,$$

$$a_{44} = 0.375 \cdot d^2 \cdot C_\delta \cdot \sum \delta_{zk}^{0.5} \cdot \cos^2 \Psi_k .$$

2. Angular contact ball bearing loaded with axial F_z and radial F_y forces.

In this case of loading the set of equations (1), [1] takes a form as follows:

$$\begin{cases} \dfrac{F_y}{C_\delta} = s_{22} \cdot f_y^{1.5} + s_{23} \cdot f_z^{1.5}; \\ \dfrac{F_z}{C_\delta} = s_{32} \cdot f_y^{1.5} + s_{33} \cdot f_z^{1.5}, \quad s_{ij} = \sum_k \sigma_{ij}^{(k)}. \end{cases} \tag{2}$$

Either equation of this set easily leads to well known expression for getting a zone of bearing loading ε, if taking to consideration simplified relationships for definition of coefficients λ_k, χ_k [2] if the bearing rolling elements contact angle is considered to be constant:

$$\lim_{\beta_k \to \beta_{0k}} \lambda_k = \sin^{2.5} \beta_{0k}; \qquad \lim_{\beta_k \to \beta_{0k}} \chi_k = \cos^{2.5} \beta_{0k};$$

Then $\sigma_{22}^{(k)} \cdot f_y^{1.5} + \sigma_{23}^{(k)} \cdot f_{23}^{(k)} = 0$ and $\cos\varepsilon = -\dfrac{f_z \cdot tg\beta_0}{f_y}$.

Set of equations (2) is the thing to solve and find f_y, f_z with the iteration process using relationships

$$f_y = \left[\frac{s_{23} \cdot F_z - s_{33} \cdot F_y}{C_\delta \cdot (s_{32} \cdot s_{23} - s_{22} \cdot s_{33})} \right]^{\frac{2}{3}}; \tag{3}$$

$$f_z = \left[\frac{\left(\dfrac{F_z}{C_\delta} - s_{32} \cdot f^{1.5} \right)}{s_{33}} \right]^{\frac{2}{3}}.$$

Coefficients s_{ij} $(i, j = 1,2,3)$ can be found through preceding iterations using expressions (2) as well as those presented in [1], [2]. Differentiating equation (2), one finds main and cross stiffness of the bearing:

$$C_{22} = \frac{\partial F_y}{\partial f_y} = 1.5 \cdot C_\delta \cdot s_{22} \cdot f_y^{0.5}; \quad C_{33} = \frac{\partial F_z}{\partial f_z} = 1.5 \cdot C_\delta \cdot s_{33} \cdot f_z^{0.5};$$

$$C_{23} = \frac{\partial F_y}{\partial f_z} = 1.5 \cdot C_\delta \cdot s_{23} \cdot f_z^{0.5}; \quad C_{32} = \frac{\partial F_z}{\partial f_y} = 1.5 \cdot C_\delta \cdot s_{32} \cdot f_y^{0.5} \qquad (4)$$

Results of calculation of bearing No 46207 are presented in table 1, the bearing loaded with external forces $F_y = 2$ kN, $F_z = 3$ kN. It's seen from there that the process of iteration is converged fast enough. For deformation f_z, the difference between the third approximation and that second is as much as just 0.3%, with practically equal approximations for deformation f_y beginning from the second iteration.

Table 1.**Calculation of bearing No 46207 parameters at combined loading. ($F_y = 2$ kN, $F_z = 3$ kN).**

Iteration number	Coefficients s_{ij}				f_y, μm	f_z, μm
	s_{22}	s_{23}	s_{32}	s_{33}		
1	3.924	-0.0307	-0.0664	1.981	12.08	24.77
2	3.967	-0.0257	-0.0553	1.910	11.97	25.35
3	3.962	-0.0254	-0.0548	1.918	11.97	25.27
4	3.963	-0.0253	-0.0548	1.917	11.97	25.28
5	3.963	-0.0253	-0.0548	1.917	11.97	25.28

Fig. 2 shows calculated relationships between bearing 46207 inner ring displacements f_y, f_z and external force F_y, with Fig. 3 to evince distribution of normal forces acting on rolling elements P_k at invariable axial load F_z = 3 kN. For making a comparison, the diagrams also present results gotten with use of the method described in [3] (see stroke lines). Moreover, stroke-dotted line on Fig. 2 shows the $f_y(F_y)$ relationship obtained with use of formulas recommended in [4] where forces and deformations were calculated with an experimental correction factor entered, to take into account the fact that real radial displacements f_y of the bearing inner ring are less than that calculated.

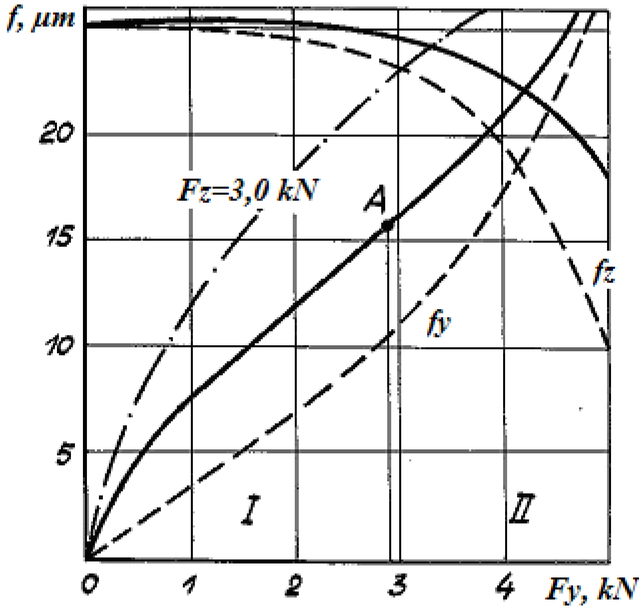

Fig. 2. Bearing 46207 inner ring displacements.

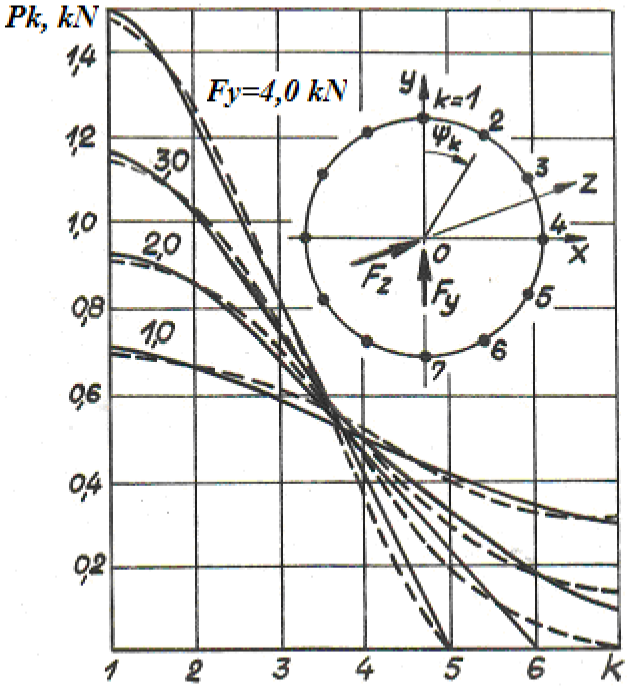

Fig. 3. Loads distribution in bearing 46207

It's well seen from Fig. 2 and Fig. 3 that while calculated loads P_k are almost coinciding, displacements somewhat differ. One should note that the relationship $f_y(F_y)$ found with the way proposed is located between and has a bend point dividing the F_y loading area into two zones. Increase of radial stiffness in that 1st and decrease of that stiffness in the 2nd one is conditioned by change of number of loaded rolling elements when increasing external load F_y. As one more example, table 2 shows results of calculation of normal reactions P_k and contact deformations $\delta_k = (P_k / C_\delta)^{\frac{2}{3}}$ taking place in bearing 2076083 loaded with external forces F_y = 5 N and F_z = 10 N. For comparison, these factors obtained in [5] are also presented in brackets there.

Table 2.**Reactions and deformations in bearing 2076083 at external loads of F_y = 5 N and F_z = 10 N.**

Angle Ψ_k ,^{0}	0	60	120	180	240	300
Reactions P_k , H	8.5 (8.1)	7.2 (7.2)	5.9 (5.6)	4.7 (4.8)	5.9 (5.6)	7.2 (7.2)
Contact deformations δ_k , µm	2.28 (2.20)	2.06 (2.04)	1.80 (1.72)	1.55 (1.56)	1.80 (1.72)	2.06 (2.04)

Conclusions. Relationships obtained on the basis of general set of equations for multi-support rolling element bearing arrangement analysis allow to find a load distribution and mutual rings displacements taken place in separate rolling element bearings with high accuracy and without much time to spend.

REFERENCES

1. S.P.Yelizarov, S.V.Konev, V.A.Artemov. Determination of stiffness characteristics of roller bearings when calculating multi-support rolling element bearings // Black Sea Region Agrarian Bulletin. – 2009. №48. – P. 52-60.
2. S.P.Yelizarov, V.A.Artemov, O.Ya.Savchenko. Determination of stiffness characteristics of ball bearings when calculating multi-support rolling element bearings // Black Sea Region Agrarian Bulletin. – 2008. №45. – P. 43-47.
3. Kovalev M.P, Narodetskyi M.Z. Calculation of high-precision bearings. – M.: Mechanical engineering. – 1980. – 373 p.
4. Beiselman R.D., Tsypkin B.V., Perel L.Ya. Rolling element bearings : Reference book. – M.: Mechanical engineering. – 1975. – 574 p.
5. Instrument ball bearing: Reference book / Edited by K.N.Yavlenskyi, V.N.Naryshkin, E.E.Chaadayeva. M.: Mechanical engineering.– 1981.– 351 p.

ИНЖЕНЕРНЫЙ МЕТОД РАСЧЕТА ОДНОРЯДНЫХ УПОРНЫХ И РАДИАЛЬНО-УПОРНЫХ ПОДШИПНИКОВ КАЧЕНИЯ ПРИ КОМБИНИРОВАННОМ НАГРУЖЕНИИ

Елизаров С.П., Липин А.П.

Ключевые слова: подшипник качения, распределение усилий, угол нагруженной зоны, осевое и радиальное перемещения кольца, угол контакта, итерационный процесс, контактные деформации.

Резюме

Получены зависимости, дающие возможность определять смещения колец и распределение нагрузок в одиночных однорядных упорных и радиально-упорных подшипниках качения с необходимой точностью и небольшими затратами времени.

ENGINEERING METHOD OF ANALYSIS OF SINGLE ROW THRUST AND ANGULAR CONTACT BALL BEARINGS AT COMPLEX LOADING

Yelizarov S.P., Lipin A.P.

Key words: rolling element bearing, load distribution, thrust bearing, loading zone, angular contact bearing, ring axial/radial displacement, contact angle, iteration process, contact reactions, contact deformations.

Summary

Relationships are obtained for fast and accurate determination of rings displacements as well as a load distribution within separate single row thrust and angular contact ball bearings at complex loading.