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Theoretical justification of the operating modes of periodic activity of vegetable oil purification

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Abstract: A mathematical model has been developed that describes the process of purification of vegetable oils using physical methods. The obtained mathematical model determines the mode of operation of the main parameters of the machine for the purification of vegetable oils, depending on the dispersed composition of the impurities and the type of vegetable oils. Based on the developed mathematical model, the recommended rotor speed and the time to remove the impurities in a centrifuge for the maximum removal of the suspended particles from different types of vegetable oils can be calculated. The response surfaces show the combined effect of the particle density and the rotational speed of the centrifuge's rotor on the impurity removal rate and the impurity removal time in sunflower and rapeseed oil. The obtained theoretical data can be used, in practice, in setting the basic parameters of the machine and selecting the centrifuge's different modes of operation with a periodic action in the purification of various vegetable oils.

Keywords: centrifuge; cleaning process; mathematical model; purifying; sunflower oil

In the purification of vegetable oils in agricultural production, centrifugation is one of the most important processes, which cleans the mechanical impurities of small dimensions – about 10–15 μm (Shokrian et al. 2018). Such cleaning has a beneficial effect on the shelf life and organoleptic properties of vegetable oils. As shown by the research in Dong et al. (2012), three-phase solid-liquid-liquid flow modelling was performed for multiphase complex fluid flow motion in a centrifugal separating machine.

This centrifugal device is designed to separate the three-phase flow, the two-phase liquid (water and olive oil) and the single-phase solid materials (olive pomace). The results include the velocity and pressure distribution for the solid phase flow (Harutyunyan et al. 2004; Pogosyan et al. 2008; Khmelev 2010; Bredikhin et al. 2017). However, this mathematical description describes the separation of coarse particles from the solid phase.

In the work of Osadchuk et al. (2020), a mathematical model of a horizontal vibrating centrifuge was created to improve the stability of the centrifuge. The results of the research show that the reduction in the attenuation coefficient of a system can reduce the amplitude of its oscillation, and increase the operating stability of the horizontal vibrating centrifuge. This mathematical description refers to the stability of the centrifuge itself. The paper of Zhao et al. (2007) describes the technology of a tube centrifuge's operation. Based on a simplified physical model, a mathematical model of the two-phase separation of solid and liquid is presented using a single-phase continuous equation.

The work of Osadchuk and Streltsov (2015) describes the mathematical modelling of the processes of the centrifugal separation of suspensions. The problem of finding the trajectories of solid-phase particles is considered. A system of equations

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is drawn up and solved, which determine the motion of solid particles in a liquid stream. An experimental and numerical study of the particle motion in the presented centrifuge rotors was performed using the example of separating a low-concentration aqueous suspension of microcrystalline cellulose.

Based on this, it can be concluded that, in the field of purification of vegetable oils by physical methods, there are practically no mathematical and relevant physical models to describe the processes of centrifugation. The speed of movement of the suspended particles and the removal time of these particles inside the centrifuge determine the properties of the impurities in the purification process. Therefore, this study is relevant for studying the dynamics of a vegetable oil inside the centrifuge.

MATERIAL AND METHODS

In the periodic operation of a centrifuge, the oil fills the working area before the rotor starts to rotate. During operation, the same liquid particles move, for which the value of the vertical velocity V_Z can be neglected. In this case, the application of the velocity field is reduced to solving a plane problem with two coordinates: radial r and angular φ. The boundary conditions can be recorded as following Equation (1):

$$\begin{aligned} &\partial_{\omega}\left(R_1,\omega\right)=\omega R_1 \\ &\partial_{r}\left(R_2,\omega\right)=0 \end{aligned} \tag{1}$$

where: R_1 – the radius of the end of the propeller; R_2 – the radius at the base of the propeller; $R_1 < r < R_2$ – adjacent area between adjacent fins; $0 < r < R_1$ – the single-connected area of the work area; ω – the rotor's angular speed of rotation.

The first boundary condition is written under the assumption that the thickness of the fin (δ) is negligibly small compared to step h, where [Equation (2)]:

$$h=2R_1 sin\frac{\pi}{n} \tag{2}$$

where: n – is the number of blades.

If this assumption is not accepted, then use Equation (3):

$$\cup_{\varphi}\left(R_1,\varphi\right)=\left(1-\frac{\delta}{h}\right)\omega R_1 \tag{3}$$

We write the third boundary condition for the areas between the adjacent fins, assuming the inequality $L < R_1$. If the width of the impeller L is significantly less than the radius of the working area, then when $R_1 < r < R_2$, you can record the average radial velocity relative to the movement using Equation (4):

$$\bar{\vartheta}_r\left(r,\varphi\right)=\frac{\left(R_2-R_1\right)}{\tau_0} \tag{4}$$

where: τ_0 – the average time for moving the oil particle across the width of the fin; R_1 – the radius of the end of the propeller; R_2 – the radius at the base of the propeller.

Because crude oil contains a dispersed material, the particles of which settle on the inner surface of the work area, a problem (or task) arises when moving particles in the area of the moving boundaries (Stefan's task).

In this case, there is a time function $R_2(\tau)$, which determines the process of filling the inter-fin space with dispersed impurities contained in the crude oil.

The impurity particle moves in the direction of the side walls can be calculated using Equation (5):

$$\vartheta_r=\frac{\rho\delta^2\vartheta_{\varphi}^2}{18\mu r} \tag{5}$$

where: ϑ_r – the radial velocity of the particle; ϑ_{φ} – the peripheral speed of the oil; μ – the dynamic viscosity of the oil; ρ – particle density; δ – the equivalent particle diameter.

Knowing the probable density of the dispersed composition of the impurities, it is possible to calculate the effective operating period of the rotary machine using Equation (6). However, it is necessary to know the velocity distribution ϑ_{φ}. We find the velocities ϑ_{φ} from the continuity and Navier-Stokes equations under the following conditions:

$$\begin{aligned} &\frac{\partial\vartheta_r}{\partial_{\varphi}}=0 \qquad \frac{\partial\vartheta_r}{\partial Z}=0 \\ &\vartheta_z=0 \qquad \frac{\partial\vartheta_z}{\partial Z}=\frac{\partial\vartheta_z}{\partial r}=\frac{\partial\vartheta_z}{\partial\varphi}=0 \end{aligned} \tag{6}$$

The system of equations under these conditions takes the form shown in following Equations (7–9):

$$\frac{\partial\vartheta_r}{\partial r}=\frac{\vartheta_r}{r}=0 \tag{7}$$

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$$\rho\left(\vartheta_r\frac{\partial\vartheta_r}{\partial r}-\frac{\vartheta_\varphi^2}{r}\right)=-\frac{\partial\rho}{\partial r}+\mu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\vartheta_r\right)\right) \tag{8}$$

$$\rho\left(\vartheta_r\frac{\partial\vartheta_r}{\partial r}+\frac{\vartheta_r\vartheta_\varphi}{r}\right)=\mu\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\vartheta_\varphi\right)\right);\ \frac{\partial\rho}{\partial Z}=\rho g \tag{9}$$

The system (8) was solved in the folowing order: From the Equation (9) it can found ϑ_r; knowing ϑ_r we can find ϑ_φ from the Equation (9); knowing ϑ_r and ϑ_φ, we can find ρ from the Equation (8).

It was applied the Equation (10):

$$\vartheta_r=\frac{A}{r} \tag{10}$$

where: A – const.; ϑ_r (0, φ) = 0; then $\vartheta_r(r, \varphi) = 0$.

Therefore, Equation (9) is simplified to Equation (11):

$$\frac{\partial\vartheta_\varphi}{\partial r}+\frac{\vartheta_\varphi}{r}=B \tag{11}$$

where: $\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\vartheta_\varphi\right)\right)=0;\ \frac{\partial}{\partial r}\left(r\vartheta_\varphi\right)=Br;\ B-\text{const}.$

A simple first-order linear equation is obtained. Let's write down its general Equation (12):

$$\vartheta_\varphi=\frac{Br}{2}+\frac{C}{r} \tag{12}$$

where: C – const.

Since the speed is limited at $r = 0$, then $C = 0$ and taking the boundary condition into account we get Equations (13):

$$\begin{aligned}\vartheta_\varphi\left(R_1\right)&=\omega R_1\\ \vartheta_\varphi\left(r\right)&=\omega r\end{aligned} \tag{13}$$

Thus, ignoring the vertical component of the oil velocity, the motion Equation (13) can be obtained according to the law of rotation of the rigid body, where ω is the angular velocity of the propeller. From Equations (11–12), we find the impurity particle's speed of approach to the fin using Equation (14):

$$\vartheta_r\left(r,\delta\right)=\frac{\rho\delta^2\omega^2}{18\mu}\times r \tag{14}$$

where: ρ – particle density; δ – the equivalent particle diameter; ω – the rotor's angular speed of rotation; μ – the dynamic viscosity of the oil.

Knowing the velocity ϑ_r, we can determine the time of movement (τ) from the fixed radial coordinate R_1 using Equation (15):

$$\tau=\int_{r_0}^{R_1}\frac{dr}{u_r} \tag{15}$$

It was applied the Equation (16):

$$\tau_\delta=\frac{18\mu}{\rho\delta^2\omega^2}\int_{r_0}^{R_1}\frac{dr}{r}=\tau_\delta\ln\frac{R_1}{r_0} \tag{16}$$

Equation (16) only makes sense for $r_0 > 0$. This is due to the fact that the particles near the axis of rotation have such a low radial velocity that they can be neglected. Then the value $r_0 > 0$ must be selected, under which two following conditions are met.

The share of particles that have a radial coordinate $r < r_0$ is not greater than the erroneous estimate of their number in the working area.

The time for moving the particle with a confidence interval with a diameter $\delta_{\min}$ does not exceed the period of its stay in the working area.

The value of r_0 in Equation (17) can now be limited by the interval $r < r_0 < R_1$. Thus, the time function for reaching the coordinate $r = R_1$ of a particle with diameter δ_2 having an arbitrary coordinate r at time $\tau = 0$ is determined in Equation (17):

$$\tau_\delta\left(r\right)=\tau_\delta\ln\frac{R_1}{r},r_1\le r\le R_1 \tag{17}$$

where: $\tau_\delta=\frac{18\mu}{\rho\delta^2\omega^2}$.

It was recorded the average time to reach the limit $r = R_1$ of the particles with a given dispersed composition with probable density distribution of diameter $f(\delta)$ [Equation (18)].

$$\overline{\tau}=\frac{1}{\left(\delta_{max}\delta_{min}\right)\left(R_1-r_1\right)}\int_{r_1}^{R_1}\int_{\delta_{min}}^{\delta_{max}}f\left(\delta\right)\tau_\delta\left(\ln\frac{R_1}{r}\right)d\,\vartheta dr \tag{18}$$

We transform this formula for the confidence interval $\delta_1 < \delta < \delta_2$ to Equation (19):

$$\overline{\tau}=\frac{18\mu}{\rho\omega^2\left(\delta_2-\delta_1\right)\left(R_1-r_1\right)}\int_{\delta_1}^{\delta_2}\frac{f\left(\delta\right)}{\delta^2}d\vartheta\int_{r_1}^{R_1}\left(\ln\frac{R_1}{r}\right)dr \tag{19}$$

Integrating within $r_1 < r < R_1$, we obtain Equations (20–21):

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$$\int_{r_1}^{R_1}\left(ln\frac{R_1}{r}\right)dr = R - r_1 - r_1\ln\frac{R_1}{r_1} \tag{20}$$

$$\bar{\tau} = \frac{18\mu}{\rho\omega^2\left(\delta_2-\delta_1\right)}\left(1-\frac{r_1}{R_1-r_1}\ln\frac{R_1}{r_1}\int_{\delta_1}^{\delta_2}\frac{f(\delta)}{\delta^2}d\delta\right) \tag{21}$$

Finally, we will record Equation (22):

$$\bar{\tau} = \frac{18\mu M_{-2}}{\rho\omega^2\left(\delta_2-\delta_1\right)}\left(1-\frac{r_1}{R_1-r_1}\ln\frac{R_1}{r_1}\right) \tag{22}$$

where: M_{-2} – initial negative moment of the second order of the distribution of diameters of the particles suspended in oil.

Using the obtained equations, we will make a theoretical calculation of the rate and time of precipitation of the suspended particles in the centrifuge with different types of oils and the dispersed composition of impurities.

RESULTS AND DISCUSSION

The modelling of centrifugation processes includes studying the optimal operating modes with an assessment of the main parameters: reliability, durability, manufacturability and efficiency of the centrifuge used in the precipitation processes for the separation of suspensions. The authors of most papers (Samarskii and Mikhailov 2001; Minkov et al. 2009; Proshin and Burkov 2010) devoted time to the construction of mathematical models of the processes and devices for separating inhomogeneous suspensions using the terms of ordinary differential equations to describe the objects. Often, the technical system is a set of many subsystems that depend on each other and on random external factors.

An analytical study of the processes that occur during the operation of devices, such as centrifuges, is often impossible. In this case, stochastic simulation models are the most effective. Studies of processes with characteristics that change at random moments are presented in many works. Systematic research into the development of an apparatus for centrifuging slurries by precipitation began in the mid-1960s. However, the high cost and complexity of the experiments do not allow for regular and comprehensive research. Research is currently underway to develop a centrifuge for slurry precipitation processes. Thus, in the works of Shiryaev (1998) and Pavlova et al. (2008), the influence of the design parameters on the operating modes of the centrifuges is considered. The works of Kuzin (2009) and Kuzin (2010) are devoted to determining the relationship between the design and technological parameters of the centrifuge processes and the radius of the inner bowl of the centrifuge collector and the angular velocity of the rotation when collecting sludge, taking the possibility of waste treatment precipitation into account.

However, the centrifuges used today have shortcomings in terms of the reliability, regularity and stability of sludge collection, the accuracy of achieving a certain amount of collection, the elimination of which requires significant financial resources. In addition, the problems of estimating the optimal angular velocity and material selection when scaling up deposition processes in radiation-hazardous industries, where waste management issues also need to be taken into account, remain unresolved. The need to reduce the cost of developing a centrifuge and the lack of mathematical models, including structural, technological, economic components, determines a significant need to develop such models.

To calculate the impurity removal rate, we used the densities and dynamic viscosities of sunflower oil and rapeseed oil. The diameter of the suspended particle and its density were set. The particle density and rotor speed were changed discretely. The results obtained are shown in Figure 1 and Figure 2.

Through the surfaces of the two responses in Figure 1 and Figure 2, the joint influence of the particle density (ρ) and rotations per min (rpm) of the centrifuge rotor (*n*), on the rate of removal of impurities (ϑ) in sunflower and rapeseed oil is shown.

With an increase in the particle density (ρ) (from 1 400 $\text{kg}{\cdot}\text{m}^{-3}$ to 2 400 $\text{kg}{\cdot}\text{m}^{-3}$) and the rotations per min (rpm) of the centrifuge rotor (*n*) (from 2 400$\cdot\text{min}^{-1}$ to 3 400$\cdot\text{min}^{-1}$), the rate of removal of impurities ϑ ($\text{m}{\cdot}\text{s}^{-1}$) increases in both cases.

The removal of impurities (ϑ) in the sunflower oil reaches its maximum (about 6×10^{-5} $\text{m}{\cdot}\text{s}^{-1}$) at an rpm of 3 400$\cdot\text{min}^{-1}$ of the centrifuge rotor (*n*) and a particle density (ρ) of 2 400 $\text{kg}{\cdot}\text{m}^{-3}$ (Figure 1). When purifying rapeseed oil, the maximum (about 8×10^{-5} $\text{m}{\cdot}\text{s}^{-1}$) is achieved at a rpm of 3 200$\cdot\text{min}^{-1}$ of the centrifuge rotor (*n*) and a particle density (ρ) of 2 400 $\text{kg}{\cdot}\text{m}^{-3}$ (Figure 2). Also, due to the physico-chemical composition of the studied oils, the high speed movement of the impurities during the cleaning of rapeseed oil can be noted.

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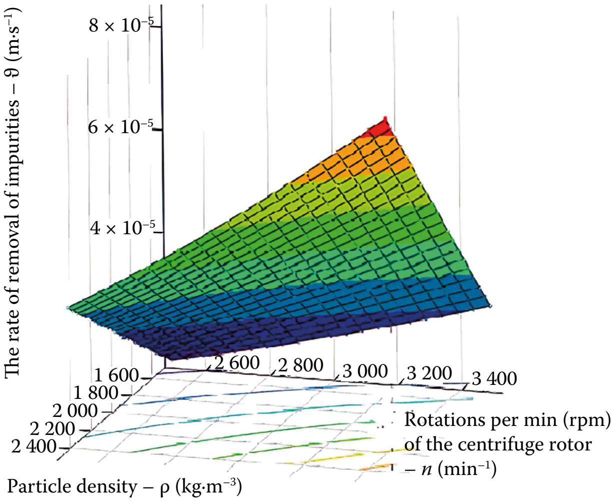

Figure 1. Influence of the particle density (ρ) and rpm of the centrifuge rotor (*n*) on the rate of removal of the impurities (ϑ) in the sunflower oil

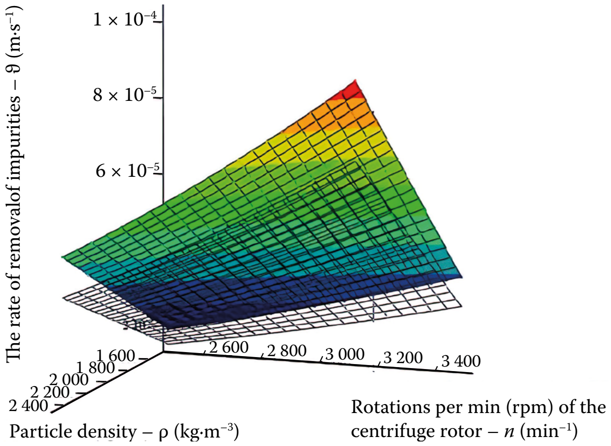

Figure 2. Influence of the particle density (ρ) and rpm of the centrifuge rotor (*n*) on the rate of removal of the impurities (ϑ) in the rapeseed oil

Using the proposed mathematical model, the removal time of the impurities in the sunflower and rapeseed oils in the centrifugal field was calculated. The obtained results are shown in Figure 3 and Figure 4.

Through the surfaces of the responses in Figure 3 and Figure 4, the combined effect of the particle density (ρ) and the rotations per min (rpm) of the centrifuge rotor (*n*) on the time for removal of impurities (τ) in sunflower and rapeseed oil is shown.

With an increasing particle density – ρ (from 1 400 kg·m^{-3} to 2 400 kg·m^{-3}) and the rotations per min (rpm) of the centrifuge rotor – *n* (from 2 400·min^{-1} to 3 400·min^{-1}), the time to remove the impurities τ (s) when purifying sunflower and rapeseed oil decreases. The optimal time for the removal of impurities – τ (s) when purifying sunflower oil is 6 500 s at an rpm of 3 400·min^{-1} of the centrifuge rotor (*n*) (Figure 3). When purifying the rapeseed oil, the optimal time for removing impurities τ (*s*) is 4 500 s at an rpm of 3 200·min^{-1} of the centrifuge rotor (*n*) (Figure 4). Due to the specific physico-chemical composition of rapeseed oil, the time to remove the impurities is shorter than that of sunflower oil.

According to the general laws of physics, we can conclude that the proposed mathematical description of the calculation of the velocity of the suspended particles and the time for their removal in a centrifugal field is correct.

The obtained theoretical data can be used, in practice, in setting the basic parameters of the machine

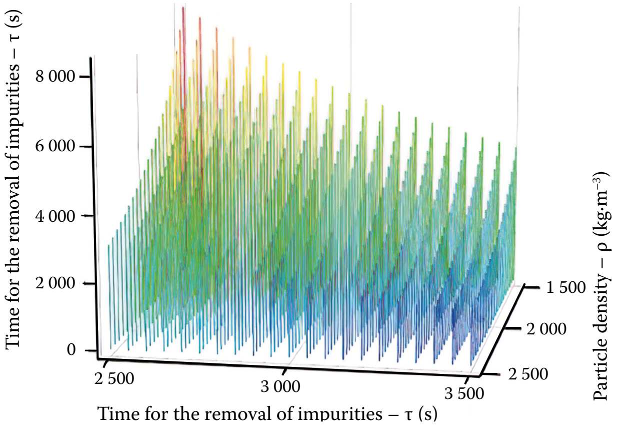

Figure 3. Influence of the particle density (ρ) and rpm of the centrifuge rotor (*n*) on the time to remove the impurities (τ) in the sunflower oil

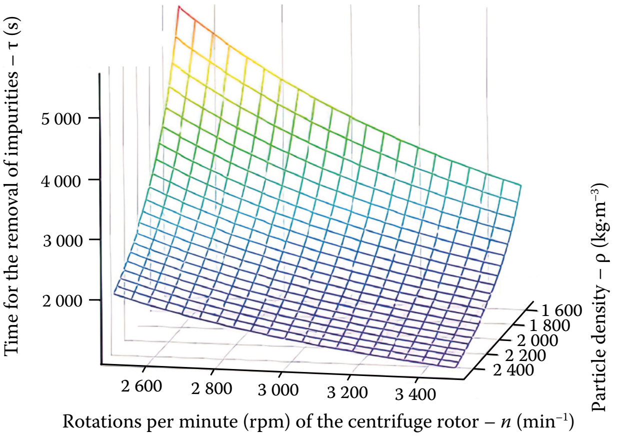

Figure 4. Influence of the particle density (ρ) and rpm of the centrifuge rotor (*n*) on the time to remove the impurities (τ) in the rapeseed oil

and selecting different modes of operation of the centrifuge with a periodic action in the purification of various vegetable oils. In our case, for the purification of sunflower oil, we recommend an rpm of 3 400·min^{-1} of the centrifuge rotor (n) with the periodic action and 6 500 s for the time to remove the impurities. For the purification of rapeseed oil, we recommend an rpm of 3 200·min^{-1} of the centrifuge rotor (n) and 4 500 s for the time to remove the impurities.

CONCLUSION

A mathematical model for the centrifugation of vegetable oils has been developed to remove the accompanying substances in the cleaning of vegetable oils by physical methods. Based on the obtained mathematical model, the optimal mode of operation of the main parameters of the machine for cleaning vegetable oils can be determined.

Depending on the dispersed composition of the impurities and the type of vegetable oils, based on the developed mathematical model, the recommended rpm of the centrifuge rotor and the time to remove the impurities in a centrifuge can be calculated for the maximum removal of suspended particles from different types of vegetable oils.

When purifying sunflower oil, we recommend a rpm of 3 400·min^{-1} of the centrifuge rotor with the periodic action and 6 500 s for the time to remove the impurities. When purifying rapeseed oil, we recommend an rpm of 3 200·min^{-1} of the centrifuge rotor and 4 500 s for the time to remove the impurities.

The obtained theoretical data can be used, in practice, in setting the basic parameters of the machine and selecting the different modes of operation of the centrifuge with a periodic action in the purification of various vegetable oils.

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